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Quasi Dα- Normal Spaces, πGDα-Closed Sets and Some Functions

Abstract

In aim this paper, we introduce a new concept of quasi-normal spaces called quasi Dα-normal spaces and obtain characterizations and preservation theorems of quasi Dα-normal. The notion can be applied for investigating many other properties.

Keywords: Dα-closed, Dαg-closed πgDα-closed, Dα-open Dαg-open,πgDα-open sets, πgDα-closed, almost πgDα-closed, πgDα-continuous and almost πgDα-continuous functions, Dα-normal spaces, mildly Dα-normal spaces and quasi Dα-normal spaces.

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Introduction

In this paper, we introduce the notion of Dαg-closed, Dαg-open, πgDα-closed,πgDα-open sets,πgDα-closed, almost πgDα-closed, πgDα-continuous and almost πgDα-continuous functions and its properties are studied. Further we introduce a new concept of quasi-normal spaces called quasi Dα-normal spaces and obtain characterizations and preservation theorems of quasi Dα-normal.

Aim of the Study

In aim this paper, we introduce a new class of sets called Dαg-closed, πgDα-closed sets and its properties are studied and we introduce a new concept of quasi-normal spaces called quasi Dα-normal spaces by using Dα-open sets due to Sayed and Khalil[11] in topological spaces and obtained several characterization and preservation theorems for quasi Dα-normal spaces. We insure the existence of utility for new results using separation axioms in topological spaces which is separate on a known separation axioms in topological spaces.

Review of Literature

The notion of quasi normal space was introduced by Zaitsev[13]. Dontchev and Noiri[2] introduce the notion of πg-closed sets as a weak form of g-closed sets due to Levine [**6**]. By using πg-closed sets, Dontchev and Noiri [**2**] obtained a new characterization of quasi normal spaces. Sayed and Khalil [**11**] introduced the concept of Dα-closed sets and discuss some of their basic properties. Recently, Reena et al. [**8**] introduced the concepts of quasi b^+-normal spaces in topological spaces by using b^+ open sets in topological spaces and obtained some characterizations of such spaces.

Preliminaries

Definition

A subset A of a topological space X is called.

Regular Closed [12]) If A = Cl(Int(A)).

Generalized Closed [4] (Briefly, g-closed) if Cl(A) $\subset$ U whenever A $\subset$ U and U is open in X .

πg-closed [2] If Cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.

α-closed [7]

If Cl(Int(Cl(A))) $\subseteq$ A .αg-closed [5]

If α-Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is in X.

πgα-closed [1] If α-Cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.

The finite union of regular open sets is said to be **π-open**. The complement of π-open set is said to be **π-closed** set. The complement of regular closed (resp. g-closed, π-open, πg-closed, α-closed, αg-closed, πgα-closed) set is said to be **regular open** (resp. **g-open**, **π-open**, **πg-open**, **α-open**, **αg-open** , **πgα-open**) sets. The intersection of all g-closed sets containing A is called the **g-closure of A** [**3**] and denoted by **Cl*(A)**, and the **g-interior of A** [**9**] is the unionof all g-open sets contained in A and is denoted by **Int*(A).**

Definition

A subset A of a topological space X is called,

Dα-closed [**11**] If Cl*(Int(Cl*(A))) $\subseteq$ A .

Dαg-closed If $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $\subseteq$ Uwhenever A $\subseteq$ U, and U is open in X.

πgDα-closed If $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.

The complement of Dα closed (resp. Dαg-closed, πgDα-closed) sets is said to be **Dα-open** (resp. **Dαg-open**, **πgDα-open**).The intersection of all Dα-closed subsets of X containing A (i.e. super sets of A) is called the **Dα-closure of A** and is denoted by$\mathbf{Cl_{\alpha}^{D}}$**(A)**. The union of all Dα-open sets contained in A is called **Dα-interior of A** and is denoted by $\mathbf{Int_{\alpha}^{D}}$(**A**).The family of all Dα-open (resp. Dα-closed) sets of a space X is denoted by **DαO(X)** (resp. **DαC(X)**).

Theorem [**11**].

Let X be a topological space. Then

1. Every α-closed subset of X is Dα-closed.
2. Every g-open subset of X is Dα-open.

We have the following implications for the properties of subsets.

$$\begin{array}{ccccc}
\text{closed} & \Rightarrow & \text{g-close} & \Rightarrow & \pi\text{g-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\alpha\text{-closed} & \Rightarrow & \alpha\text{g-closed} & \Rightarrow & \pi\text{g}\alpha\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\text{D}\alpha\text{-closed} & \Rightarrow & \text{D}\alpha\text{g-closed} & \Rightarrow & \pi\text{gD}\alpha\text{-closed}
\end{array}$$

Where none of the implications is reversible as can be seen from the following examples

Example

Let X = {a, b, c , d } and τ = { ϕ, {a}, {c, d}, {a, c, d }, {d}, {a, d}, X }. Then the set A = {a} is πgα-closed set as well πgDα-closed set but not g-closed set in X .

Example

Let X = {a, b, c , d } and τ = { ϕ, {a}, {c}, {a, b}, {a, c}, {a, d}, {a, d, c }, {a, b, d}, {a, b, c}, X}. Then the set A = {a, b} is πgα-closed set as well as πgDα-closed set but not αg-closed and not Dαg-closed set in X.Since A $\subset$ {a, b, c} which is open by $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$ $\not\subset$ {a, b, c}.

Example

Let X = {a, b, c, d } and τ = { ϕ, {a}, {c, d}, {a, c, d}, {d}, {a, d}, X}. Then the set A = {c} is πgα-closed set as well as πgDα-closed set but not πg-closed set in X.

Theorem

1. Finite union of πgDα-closed sets are πgDα-closed.
2. Finite intersection of πgDα-closed need not be a πgDα-closed.
3. A countable union of πgDα-closed sets need not be a πgDα-closed.

Proof

1. Let A and B be πgDα-closed sets. Therefore $\mathrm{Cl}_{\alpha}^{\mathrm{D}}(A)\subset$ U and $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(B) $\subset$ U whenever A $\subset$ U, B $\subset$ U and U is π-open. Let A $\cup$ B $\subset$ U where U is π-open. Since $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A $\cup$ B) $\subset$ $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $\cup$ $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(B) $\subset$ U, we have A $\cup$ B is πgDα-closed.
2. Let X = {a, b, c, d} and τ = { ϕ, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}, X}. Let A = {a, b, c}, B = {a, b, d}. A and B are πgDα-closed sets. But A $\cap$ B = {a, b}$\subset${a, b} which is π-open. $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A $\cap$ B) $\not\subset$ {a, b}. Hence A $\cap$ B is not πgDα-closed.
3. Let R be the real line with the usual topology. Every singleton is πgDα-closed. But, A = {1/i : i = 2, 3, 4 ……………} is not πgDα-closed. Since A $\subset$ (0, 1) which is π-open but $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $\not\subset$ (0, 1).

Theorem

If A is πgDα-closed and A $\subset$ B $\subset$ $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) then B is πgDα-closed.

Proof

Since A is πgDα-closed, $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $\subset$ U whenever A $\subset$ U and U is π-open. Let B $\subset$ U and U be π-open. Since B $\subset$ $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A), $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(B) $\subset$ $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $\subset$ U. Hence B is πgDα-closed.

Theorem

Let A be a πgDα-closed set in X. Then $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $-$ A does not contain any non empty π-closed set.

Proof

Let F be a non empty π-closed set such that F $\subset$$\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $-$ A. Then F $\subset$ $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $\cap$ (X $-$ A) $\subset$ X $-$ A implies A $\subset$ X $-$ F where X $-$ F is π-open. Therefore $\mathrm{Cl}_{\alpha}^{\mathrm{D}}(A)\subset$ X $-$ F implies F $\subset$ ($\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A)^{C} . Now F $\subset$ $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $\cap$ ($\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A))^{C} implies F is empty.

Reverse implication does not hold.

Corollary

Let A be πgDα-closed. A is Dα-closed iff $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $-$ A is π-closed.

Proof. Let A be Dα-closed set then A = $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) implies $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $-$ A = ϕ which is π-closed.

Conversely if $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $-$ A is π-closed then A is Dα-closed.

Theorem

If A is π-open and πgDα-closed. Then A is Dα-closed hence clopen.

Proof

Let A be regular open. Since A is πgDα-closed, $\mathrm{Cl}_{\alpha}^{\mathrm{D}}$(A) $\subset$ A implies A is Dα-closed. Hence A is closed (Since every π-open, Dα-closed set is closed). Therefore A is clopen.

Theorem

For a topological space X, the following are equivalent :

1. X is extremally disconnected.
2. Every subset of X is πgDα-closed.
3. The topology on X generated by πgDα-closed sets.

Proof

(a) $\Rightarrow$ (b). Assume X is extremally disconnected. Let $A \subset U$, where U is π-open in X. Since U is π-open , it is the finite union of regular open sets and X is extremally disconnected, U is finite union of clopen sets and hence U is clopen. Therefore $\mathrm{Cl}_{\alpha}^{D}(A) \subset Cl(A) \subset Cl(U) \subset U$ implies A is πgDα-closed.

(b) $\Rightarrow$ (a). Let A be reguler open set of X. Since A is πgDα-closed by **Theorem 2.11** A is clopen. Hence X is extremally disconnected.

(b) $\Leftrightarrow$ (c) is obvious.

Lemma[**11**]

If A is a subset of X, then

1. $X - \mathrm{Cl}_{\alpha}^{D}(A) = \mathrm{Int}_{\alpha}^{D}(X - A)$.
2. $\mathrm{Cl}_{\alpha}^{D}(X - A) = X - \mathrm{Int}_{\alpha}^{D}(A)$.

Theorem

A subset A of a topological space X is πgDα-open if $F \subset \mathrm{Int}_{\alpha}^{D}(A)$ whenever F is π-closed and $F \subset A$.

Proof

Let F be π-closed set such that $F \subset A$. Since $X - A$ is πgDα-closed and $X - A \subset X - F$ we have $F \subset \mathrm{Int}_{\alpha}^{D}(A)$.

Conversely, Let $F \subset \mathrm{Int}_{\alpha}^{D}(A)$ where F is π-closed and $F \subset A$. Since $F \subset A$ and $X - F$ is π-open, $\mathrm{Cl}_{\alpha}^{D}(X - A) = X - \mathrm{Int}_{\alpha}^{D}(A) \subset X - F$. Therefore A is πgDα-open.

Theorem

If, $\mathrm{Int}_{\alpha}^{D}(A) \subset B \subset A$ and A is πgDα-open then B is πgDα-open.

Proof

Since, $\mathrm{Int}_{\alpha}^{D}(A) \subset B \subset A$ using **Theorem 2.8**, $\mathrm{Cl}_{\alpha}^{D}(X - A) \supset (X - B)$ implies B is πgDα-open.

Remark

For any $A \subset X$, $\mathrm{Int}_{\alpha}^{D}(\mathrm{Cl}_{\alpha}^{D}(A)) - A) = \phi$.

Theorem

If $A \subset X$ is πgDα-closed then $\mathrm{Cl}_{\alpha}^{D}(A) - A$ is πgDα-open.

Proof

Let A be πgDα-closed and F be a π-closed set such that $F \subset \mathrm{Cl}_{\alpha}^{D}(A) - A$. By **Theorem 2.9**

$F = \phi$ implies $F \subset \mathrm{Int}_{\alpha}^{D}(\mathrm{Cl}_{\alpha}^{D}(A) - A))$. By **Theorem 2.14**, $\mathrm{Cl}_{\alpha}^{D}(A) - A$ is πgDα-open.

Converse of the above theorem is not true.

Example

Let X = {a, b, c } and τ = { ϕ, {a}, {b}, {a, b}, X}. Let A = {b }.Then A is not πgDα-closed but $\mathrm{Cl}_{\alpha}^{D}(A) - A = \{a, b\}$ πgDα-open.

Quasi Dα-normal spaces

Definition

A topological space X is said to be **Dα-normal** (resp. **quasi Dα-normal** , **mildly Dα-normal**) if for every pair of disjoint closed (resp. π-closed, regularly closed) subsets H, K of X, there exist disjoint Dα-open sets U, V of X such that $H \subset U$ and $K \subset V$.

Example

Let X = {a, b, c, d} and τ = {ϕ, {a }, {b},{a, b}, {a, b, c }, X}. The pair of disjoint closed subsets of X are A = ϕ and B = {d}.Then Dα-closed sets in X are X, ϕ , {a}, {b}, {c}, {d}, {c, d}, {a, d}, {b, c},{a, c}, {b, d}, {a, c, d}, {b, c, d}. Also U = { b} and V = {c, d} are Dα-open sets such that $A \subset U$ and $B \subset V$. Hence X is Dα-normal but it is not normal.

Example

Let X = { a, b, c, d } and τ = { ϕ, {a}, {c}, {a, c},{ b, d}, {a, b, d}, {b, c, d}, X }. The pair of disjoint π-closed subsets of X are A = {a} and B = {c}. Also U = {a} and V = {b, c, d} are open sets such that $A \subset U$ and $B \subset V$. Hence X is quasi-normal as well as quasi Dα-normal because every open set is Dα-open set.

By the definitions and examples stated above, we have the following diagram:

$$\begin{array}{ccccc} \text{normality} & \Rightarrow & \text{quasi-normality} & \Rightarrow & \text{mild-normality} \\ \Downarrow & & \Downarrow & & \Downarrow \\ D\alpha\text{-normality} & \Rightarrow & \text{quasi } D\alpha\text{-normality} & \Rightarrow & \text{mild } D\alpha\text{-normality} \end{array}$$

Theorem

For topological space X , the following are equivalent:

a. X is quasi Dα-normal.
b. For any disjoint π-closed sets H and K, there exist disjoint Dαg-open sets U and V such that $H \subset U$ and $K \subset V$.
c. For any disjoint π-closed sets H and K, there exist disjoint πgDα-open sets U and V such that $H \subset U$ and $K \subset V$.
d. For any π -closed set H and any π-open set V containing H, there exist a Dαg-open set U of X such that $H \subset U \subset \mathrm{Cl}_{\alpha}^{D}(U) \subset V$.
e. For any π-closed set H and any π-open set V containing H, there exist a πgDα-open set U of X such that $H \subset U \subset \mathrm{Cl}_{\alpha}^{D}(U) \subset V$.

Proof

(a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c), (d) $\Rightarrow$ (e), (c) $\Rightarrow$ (d) and (e) $\Rightarrow$ (a). (a) $\Rightarrow$ (b).

Let X be quasi Dα-normal. Let H, K be disjoint π-closed sets of X. By assumption, there exist disjoint Dα-open sets U, V such that $H \subset U$ and $K \subset V$. Since every Dα-open set is Dαg-open, U,V are Dαg-open sets such that $H \subset U$ and $K \subset V$.

(b) $\Rightarrow$ (c). Let H, K be two disjoint π -closed sets. By assumption, there exists Dαg-open sets U and V such that $H \subset U$ and $K \subset V$. Since Dαg-open set is πgDα-open, U and V are πgDα-open sets such that $H \subset U$ and $K \subset V$.

(d) $\Rightarrow$ (e). Let H be any π -closed set and V be any π -open set containing H. By assumption, there exist Dαg-open set U of X such that $H \subset U \subset \mathrm{Cl}_{\alpha}^{D}(U) \subset V$. Since every Dαg-open set is πgDα-open, there exist πgDα-open sets U of X such that $H \subset U \subset \mathrm{Cl}_{\alpha}^{D}(U) \subset V$.

(c) $\Rightarrow$ (d). Let H be any π-closed set and V be any π-open set containing H. By assumption, there exist πgDα-open sets U and W such that $H \subset U$ and $X - V \subset W$. By **Theorem 2.14**, we get $X - V \subset \mathrm{Int}_{\alpha}^{D}(W)$ and $\mathrm{Cl}_{\alpha}^{D}(U) \cap \mathrm{Int}_{\alpha}^{D}(W) = \phi$. Hence $H \subset U \subset \mathrm{Cl}_{\alpha}^{D}(U) \subset X - \mathrm{Int}_{\alpha}^{D}(W) \subset V$.

(e) ⇒ (a). Let H, K be any two disjoint π-closed set of X. Then $H \subset X - K$ and $X - K$ is π -open. By assumption, there exist πg Dα-open set G of X such that $H \subset G \subset Cl^D_\alpha(G) \subset X - K$. Put $U = Int^D_\alpha(G)$, $V = X - Cl^D_\alpha(G)$. Then U and V are disjoint Dα-open sets of X such that $H \subset U$ and $K \subset V$.

Some Functions

Definition

A function $f : X \to Y$ is said to be

1. Almost closed [10](resp. almost Dα-closed , almost Dαg-closed) if f (F) is closed (resp. Dα-closed , Dαg-closed) in Y for every $F \in RC(X)$.
2. πgDα-closed (resp. almost πgDα-closed) if for every closed set (resp. regularly closed) F of X , f(F) is πgDα-closed in Y.
3. π-continuous [2] (resp. πgα-continuous[1] , πgDα-continuous) if $f^{-1}(F)$ is π-closed (resp.πgα-closed, πgDα-closed) in X for every closed set F of Y.
4. Almost continuous [10] (resp. almost π-continuous [2], almost πgα- continuous[1] , almost πgDα-continuous) if $f^{-1}(F)$ is closed (resp. π- closed, πgα-closed, πgDα-closed) in X for every regularly closed set F of Y.
5. Rc-preserving [6] if f(F) is regularly closed in Y for every $F \in RC(X)$.

From the definitions stated above, we obtain the following diagram:

closed ⇒ α-closed ⇒ αg-closed ⇒ πgα-closed
⇓ ⇓ ⇓ ⇓
al.-closed ⇒ al.Dα-closed ⇒ al. Dαg-closed ⇒al. πgDα-closed

Where al. = almost

Moreover, by the following examples, we realize that none of the implications is reversible.

Example

$X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{c\}, \{a, b, d\}\}$ and $\sigma = \{\phi, \{a\}, \{c, d\}, \{a, c, d\} \{d\}, \{a, d\}, X\}$. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function, then f is πgα-closed as well as πgDα-closed but not πg-closed. Since A= {c} is not πg-closed in (X, σ).

Example

Let $X = \{a, b, c, d\}$, $\tau = \{\phi , X, \{c\},\{a, b, d\}, \{b, d\}, \{b, c, d\}, X \}$ and $\sigma = \{\phi, X, \{a\},\{c, d\}, \{a, c, d\}, \{d\}, \{a, d\}\}$.Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then f is almost πgα-closed as well as almost πgDα-closed but not πgDα-closed. Since A = {a} is not πgDα-closed

Theorem

If f: $X \to Y$ is an almost π-continuous and πgDα-closed function, then f(A) is πgDα-closed in Y for every πgDα-closed set A of X.

Proof

Let A be any πgDα-closed set A of X and V be any π-open set of Y containing f(A). Since f is almost π-continuous, $f^{-1}(V)$ is π-open in X and $A \subset f^{-1}(V)$. Therefore $Cl^D_\alpha(A) \subset f^{-1}(V)$ and hence $f(Cl^D_\alpha(A)) \subset V$. Since f is πgDα-closed, $f(Cl^D_\alpha(A))$ is πgDα-closed in Y. And hence we obtain $Cl^D_\alpha(f(A)) \subset Cl^D_\alpha(f(Cl^D_\alpha(A))) \subset V$. Hence f(A) is πgDα-closed in Y.

Theorem

A surjection $f : X \to Y$ is almost πgDα-closed if and only if for each subset S of Y and each $U \in RO(X)$ containing $f^{-1}(S)$ there exists a πgDα-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof

Necessity, suppose that f is almost πgDα-closed. Let S be a subset of Y and $U \in RO(X)$ containing $f^{-1}(S)$. If $V = Y - f(X - U)$, then V is a πgDα-open set of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Sufficiency, Let F be any regular closed set of X. Then $f^{-1}(Y - f(F)) \subset X - F$ and $X - F \in RO(X)$. There exists πgDα-open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore, we have $f(F) \supset Y - V$ and $F \subset X - f^{-1}(V) \subset f^{-1}(Y - V)$. Hence we obtain $f(F) = Y - V$ and f(F) is πgDα-closed in Y which shows that f is almost πgDα-closed.

Preservation Theorem

Theorem

If $f : X \to Y$ is an almost πgDα-continuous, rc-preserving injection and Y is quasi Dα-normal then X is quasi Dα-normal.

Proof

Let A and B be any disjoint π-closed sets of X. Since f is an rc-preserving injection, f(A) and f(B) are disjoint π-closed sets of Y. Since Y is quasi Dα-normal, there exist disjoint Dα-open sets U and V of Y such that $f(A) \subset U$ and $f(B) \subset V$.

Now if $G = Int(Cl(U))$ and $H = Int(Cl(V))$. Then G and H are regularly open sets such that $f(A) \subset G$ and $f(B) \subset H$. Since f is almost πgDα-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint πgDα-open sets containing A and B which shows that X is quasi Dα-normal.

Theorem

If f: $X \to Y$ is π-continuous, almost Dα-closed surjection and X is quasi Dα-normal space then Y is Dα-normal.

Proof

Let A and B be any two disjoint closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π-closed sets of X. Since X is quasi Dα-normal, there exist disjoint Dα-open sets of U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Let $G = Int(Cl(V))$ and $H = Int(Cl(V))$. Then G and H are disjoint regularly open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. Set $K = Y - f(X - G)$ and $L = Y - f(X - H)$.Then K and L are Dα-open sets of Y such that $A \subset K, B \subset L, f^{-1}(K) \subset G , f^{-1}(L) \subset H$. Since G and H are disjoint, K and L are disjoint. Since K and L are Dα-open and we obtain $A \subset Int^D_\alpha(K)$, $B \subset Int^D_\alpha(L)$ and $Int^D_\alpha(K) \cap Int^D_\alpha(L) = \phi$.Therefore Y is Dα-normal.

Theorem

Let f : X → Y be an almost π-continuous and almost πgDα-closed surjection. If X is quasi Dα-normal space then Y is quasi Dα-normal.

Proof. Let A and B be any disjoint π-closed sets of Y. Since f is almost π-continuous, $f^{-1}(A)$, $f^{-1}(B)$ are disjoint closed subsets of X. Since X is quasi Dα-normal, there exist disjoint Dα-open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$.

Let G = Int(Cl(U)) and H = Int(Cl(V)). Then G and H are disjoint regularly open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. By Theorem 4.5, there exist πgDα-open sets K and L of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, so are K and L by Theorem 2.14, $A \subset \mathrm{Int}_\alpha^D(K)$, $B \subset \mathrm{Int}_\alpha^D(L)$ and $\mathrm{Int}_\alpha^D(K) \cap \mathrm{Int}_\alpha^D(L) = \phi$. Therefore Y is quasi Dα-normal.

Corollary

If f : X → Y is almost continuous and almost closed surjection and X is a normal space, then Y is quasi Dα-normal.

Proof

Since every almost closed function is almost πgDα-closed so Y is quasi Dα-normal.

Conclusion

The notion of quasi Dα-normal in topological spaces has been generalized and obtain characterizations and preservation theorems of quasi Dα-normal.

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